In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. What is $\lim_{h \to 0} \dfrac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$

(A) 1

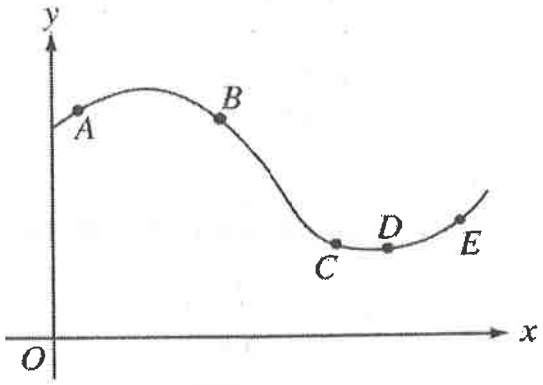
(B) $\dfrac{\sqrt{2}}{2}$

(C) 0

(D) -1

(E) The limit does not exist.

2. At which of the five points on the graph in the figure at the right are $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ both negative?



(A) A

(B) B

(C) C

(D) D

(E) E

3. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at (2, 1) is

(A) $-\dfrac{3}{2}$

(B) 1

(C) $-\dfrac{5}{14}$

(D) $-\dfrac{3}{14}$

(E) 0

4. A city is built around a circular lake that has a radius of 1 mile. The population density of the city is $f(r)$ people per square mile, where r is the distance from the center of the lake, in miles. Which of the following expressions gives the number of people who live within 1 mile of the lake?

(A) $2\pi\int_0^1 rf(r)\,dr$

(B) $2\pi\int_0^1 r(1+f(r))\,dr$

(C) $2\pi\int_0^2 r(1+f(r))\,dr$

(D) $2\pi\int_1^2 rf(r)\,dr$

(E) $2\pi\int_1^2 r(1+f(r))\,dr$

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.
(B) The graph of the function has one point of inflection and the function has two relative extrema.
(C) The graph of the function has two points of inflection and the function has one relative extremum.
(D) The graph of the function has two points of inflection and the function has two relative extrema.
(E) The graph of the function has two points of inflection and the function has three relative extrema.

6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2 \cos 3$
(B) $-2 \sin 3 \cos 3$
(C) $6 \cos 3$
(D) $2 \sin 3 \cos 3$
(E) $6 \sin 3 \cos 3$

7. The solution to the differential equation $\frac{dy}{dx} = \frac{x^3}{y^2}$, where $y(2) = 3$, is

(A) $y = \sqrt[3]{\frac{3}{4}x^4}$

(B) $y = \sqrt[3]{\frac{3}{4}x^4} + \sqrt[3]{15}$

(C) $y = \sqrt[3]{\frac{3}{4}x^4} + 15$

(D) $y = \sqrt[3]{\frac{3}{4}x^4 + 5}$

(E) $y = \sqrt[3]{\frac{3}{4}x^4 + 15}$

8. What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

9. The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $3 < t < 4$ only
(B) $t > 4$ only
(C) $t > 5$ only
(D) $0 < t < 3$ and $t > 5$
(E) $3 < t < 4$ and $t > 5$

10. $\int (x-1)\sqrt{x}\,dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$

(B) $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

(C) $\frac{1}{2}x^2 - x + C$

(D) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$

(E) $\frac{1}{2}x^2 + 2x^{\frac{3}{2}} - x + C$

11. What is $\lim_{x\to\infty} \frac{x^2-4}{2+x-4x^2}$?

(A) -2

(B) $-\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) The limit does not exist.

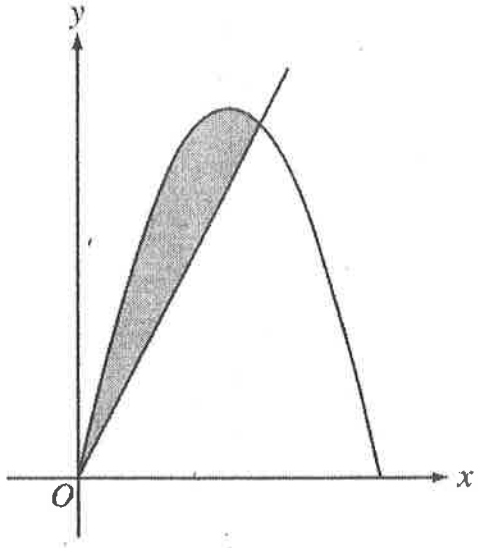


figure for # 12

12. The figure above shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

(A) $\frac{25}{6}$

(B) $\frac{9}{2}$

(C) 9

(D) $\frac{27}{2}$

(E) $\frac{45}{2}$

13. If f is a function that is continuous for all real numbers, then $\frac{d}{dx}\int_0^{x^2} f(t)\,dt =$

(A) $2xf(x^2)$

(B) $2xf(2x)$

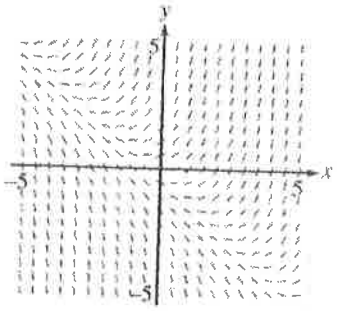
(C) $f(2x)$

(D) $f(x^2)$

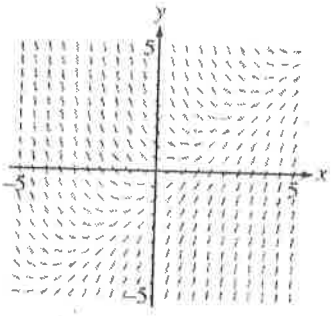
(E) $f'(x^2)$

14. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

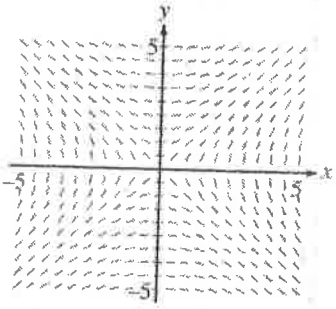
(A)



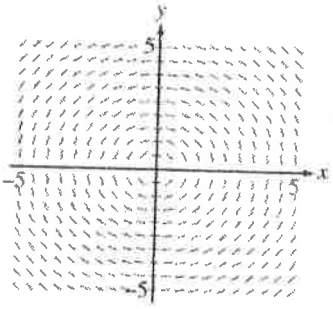
(B)



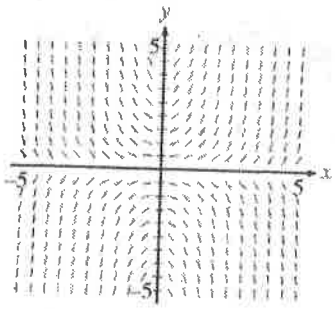
(C)



(D)



(E)



Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part B and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15. A particle travels along a straight line with a velocity of $v(t) = 3e^{(-t/2)} \sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835
(B) 1.850
(C) 2.055
(D) 2.261
(E) 7.025

16. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

(A) $\pi \int_0^1 \left(\left(3 - 2x^2\right)^2 - \left(3 - 2x\right)^2 \right) dx$

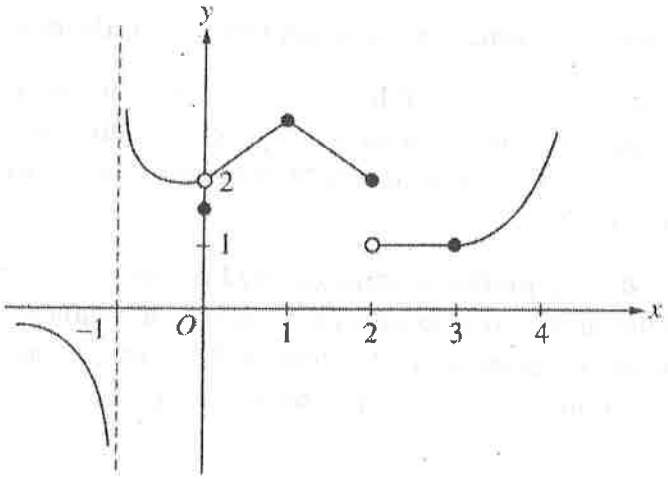
(B) $\pi \int_0^1 \left(\left(3 - 2x^2\right)^2 - \left(3 - 2x^2\right)^2 \right) dx$

(C) $\pi \int_0^1 \left(4x^2 - 4x^4\right) dx$

(D) $\pi \int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2 \right) dy$

(E) $\pi \int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2 \right) dy$

17.



The graph of a function f is shown above. If $\lim_{x \to b} f(x)$ exists and f is not continuous at b, then $b =$

(A) −1
(B) 0
(C) 1
(D) 2
(E) 3

18.

x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

Let f be a function such that $f''(x)$ for all x in the closed interval [1, 2]. Selected values of f are shown in the table above. Which of the following must be true about $f'(1.2)$?

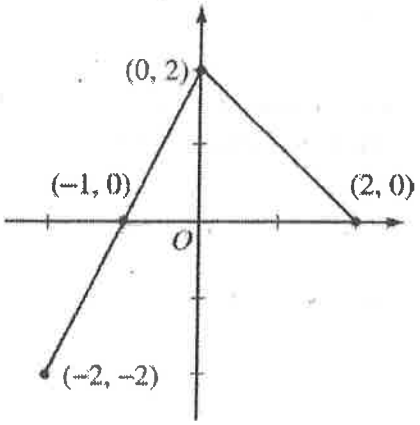
(A) $f'(1.2) < 0$
(B) $0 < f'(1.2) < 1.6$
(C) $1.6 < f'(1.2) < 1.8$
(D) $1.8 < f'(1.2) < 2.0$
(E) $f'(1.2) > 2.0$

$f'' < 0$

19. Two particles start at the origin and move along the x-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

20.



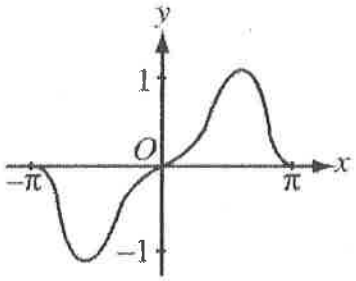
Graph of f

The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x)=\int_0^x f(t)dt$, then $g(-1)=$

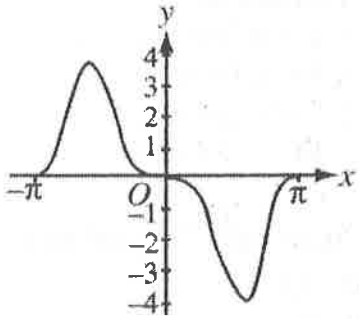
(A) −2
(B) −1
(C) 0
(D) 1
(E) 2

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi,\pi]$?

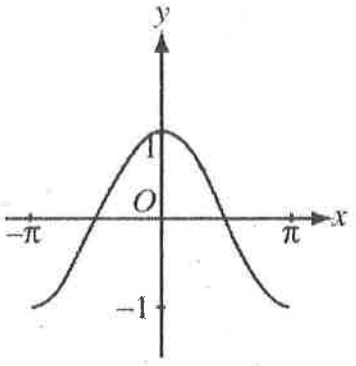
(A)



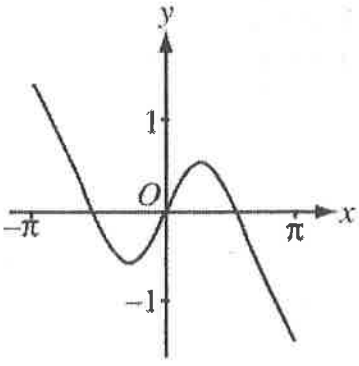
(B)



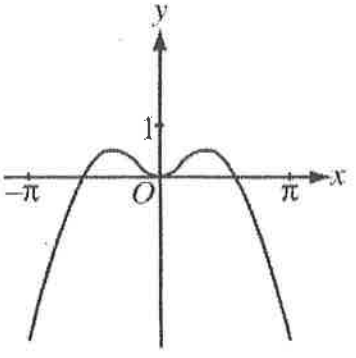
(C)



(D)



(E)



22. The base of a solid S is the semicircular region enclosed by the graph of $y = \sqrt{4 - x^2}$ and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{32\pi}{3}$

(B) $\frac{16\pi}{3}$

(C) $\frac{40}{3}$

(D) $\frac{32}{3}$

(E) $\frac{16}{3}$

23. Oil is leaking from a tanker at the rate of gallons $R(t) = 2{,}000e^{-0.2t}$ per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

24. If $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$

(A) -1.819
(B) -0.843
(C) -0.819
(D) 0.157
(E) 1.157